On the Effect of Asymmetry on Wave-length Determinations.

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It is a familiar experience in spectroscopy that the actual accuracy which is attained in precise measurements of wave-length is almost invariably disappointing, and falls far short of the precision to be expected from a consideration of the probable errors. In particular, the measurement of unsymmetrical lines, difficult in itself, is affected by displacements which depend on the limited resolving power of the spectrograph. It is no doubt well known that this effect must be operative, and it has been considered, for example, by Grebe and Bachem* in their search for the Einstein effect in the solar spectrum. But we are not aware of any precise formulation of its magnitude, although a close approximation to the apparent displacement of the maximum of an unsymmetrical line, expressed in the terms of the resolving power and the asymmetry, can be obtained from simple considerations.

We have at present no very precise knowledge of the intensity distribution curves of the unsymmetrical lines which are found in the spectra of the electric arc and of numerous other sources. But we have convinced ourselves, from an inspection of the photometric curves obtained by King and Koch† and St. John and Babcock,‡ that for the present purpose, we may assume, without serious error, that the lines may be represented by the purely artificial device of combining two error curves.

We do this by assuming that the intensity falls off from the maximum, on either side of the line, according to the law

$$I \propto e^{-kx^2}.$$

The asymmetry is reproduced by adopting different values of k on the two sides of the maximum.

It is evident that, with infinite resolving power, the positions of the true and apparent maxima will be coincident; but it will at once be seen that, with a finite resolving power, the energy at any point is gathered over a range prescribed, on the usual considerations of wave theory, by the diffraction pattern of the slit. The practical resolving power, however, is affected by other considerations, such as the grain of the plate, and, for

* 'Deutsch. Phys. Gesell. Verh.,' 21, p. 454 (1919).
† 'Astrophys. Journ.,' vol. 39, p. 213 (1914).
‡ *Ibid.*, vol. 42, p. 231 (1914).

practical purposes, we may consider that the light is gathered over a range which is equal to the actual purity of the spectrum. We proceed as follows :—

Consider a line, whose maximum intensity occurs at $x = 0$, which is broadened in the direction of negative values of x according to the intensity distribution law

$$y = e^{-kx^2},$$

and, in the direction of positive values, according to the law

$$y = e^{-k'x^2}.$$

Let the purity of the spectrum be represented by the interval 2ξ, the energy being thus gathered on either side of any point over a range, ξ. Let Q be the abscissa of any point. Then the energy, E, gathered at this point is represented, on the complete graph of the lines, by the area lying above an interval, 2ξ, bisected by the point, Q. Thus

$$\mathrm{E} = \int_{-(\xi-\mathrm{Q})}^{0} dx\, e^{-kx^2} + \int_{0}^{\xi+\mathrm{Q}} dx\, e^{-k'x^2}.$$

It is clear that Q is the value of the displacement required when it is so selected that E is a maximum, or $d\mathrm{E}/d\mathrm{Q}$ is zero.

Since Q only occurs in the limits of integration, this yields at once

$$-e^{-k(\mathrm{Q}-\xi)^2} + e^{-k'(\mathrm{Q}+\xi)^2} = 0,$$

or

$$k(\mathrm{Q}-\xi)^2 = k'(\mathrm{Q}+\xi)^2,$$

whence

$$\frac{\mathrm{Q}}{\xi} = \frac{\sqrt{k}-\sqrt{k'}}{\sqrt{k}+\sqrt{k'}} \qquad (k > k'),$$

rejecting the value which is obviously inapplicable. Thus the required displacement, Q, is

$$\mathrm{Q} = \xi\left(\frac{\sqrt{(k/k')}-1}{\sqrt{(k/k')}+1}\right).$$

It is convenient, at this point, to specify the degree of asymmetry in some simple quantitative manner. For this purpose we adopt the ratio of the two "half-widths" of the line on either side of the maximum, these being the values of x which make y equal to $\frac{1}{2}$. They are given by

$$\tfrac{1}{2} = e^{-kx^2}, \qquad \tfrac{1}{2} = e^{-k'x^2},$$

and calling them x_1 and x_2, we have

$$\frac{x_1}{x_2} = \sqrt{\frac{k'}{k}}, \qquad \frac{x_2}{x_1} = \sqrt{\frac{k}{k'}}.$$

The quotient of the greater value of x by the smaller may be called the

index of asymmetry and denoted by the symbol i. Thus, if $k > k'$, as we assumed above,

$$i = \sqrt{\frac{k}{k'}},$$

and the formula for the displacement reduces to

$$Q = \xi\left(\frac{i-1}{i+1}\right).$$

It is evident that in most spectroscopic measurements, where the accuracy of micrometer settings far transcends the resolving power, this correction is by no means inconsiderable. Thus, on a plate on which lines separated by 0·1 A. can just be resolved, the apparent displacement in the case of a line whose index of asymmetry is 3 is 0·05 Å, and it is further to be noted that this displacement is independent of the actual values of k and k'. Thus, though a line may be so narrow that no trace of asymmetry can be detected, it may nevertheless show a displacement which might in certain cases lead to entirely wrong conclusions. It is at least possible that some of the difficulties experienced in fitting a formula to a spectrum series may be due to errors arising in this manner. We are thus led to the conclusion that the general practice of measuring wave-lengths to a degree of accuracy far transcending the purity of the spectrum, depends on the assumption that the lines show no serious asymmetry. Whilst the assumption can hardly be accepted generally, it is of interest to note that the above considerations suggest a method of measuring the index of asymmetry in some cases in which the physical width of the line is exceedingly small, for it is evident that, since the displacement can never exceed the purity of the spectrum, a series of measurements made, for example, in different orders of the grating spectrum would afford material for the calculation of the asymmetry. The application of these principles to investigations such as the displacement of lines in the solar spectrum is so evident as to need no further comment.